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PRODUCT GUARANTEE COMMITMENT: BASED ON GAME THEORY APPROACHES

Abstract. *Commitment plays an extremely important role in interactions between multiple decision-makers. This paper highlights the effects of a commitment to guarantee goods with high maintenance costs under duopoly. The strategies of both producers under the Bertrand game model are analyzed when guarantee commitments are launched. Surprisingly, we find that technological innovation may result in a decrease in profits. Or the guarantee commitment deters the innovation. Commitments are discussed in the industrial organization community.*

Keywords: *Economics, game theory, industrial organization, product maintenance, commitment.*
JEL Classification: C61, C72，D4, L1

I. Introduction

Since commitment is very important in decision, commitment is extensively investigated in economics (Krueger, Lustig & Perri, 2008；Nie, 2009; Foote & Tang, 2008; Kempf and Thadden, 2013 ;Yucel, McMillan, and Richard, 2014). Zwetsloot, Aaltonen,Wybo et al. (2013) argued that commitment strategies improve the safety. There exist various types of commitments in reality and product guarantee commitment is extremely important to the whole consumer market for products with maintenance or repair costs. First, a commitment to guarantee a product can improve the confidence of consumers to buy that product (Kingshott and Pecotich, 2007; Caceres and Paparoidamis, 2007; Peysakhovich, 2014). Through quality commitment, product differentiation is efficiently promoted (Gupta, Grant and Melewar, 2008). Second, producers thereby obtain feedback information for the corresponding products to motivate and guide technological innovation. Third, guarantee commitments can change strategies of the opponents. Fourth, guarantee commitments can enhance the relationship between the producers and the consumers (Sahadev, 2008). Finally, guarantee commitments are often regulated by laws and policies (Quadrini, 2005). It is therefore important to further investigate guarantee commitments.

The relationship between guarantee and quality seems important. Much research supports a positive relationship between the service guarantee and quality. However, some research has questioned this type of relationship, as in a recent article by Heys and Hill (2006) and the references mentioned therein. Estelami, Grewal and Roggeveen (2007) focused on price-matching guarantees (PMGs) by retailers and argued that PMGs can result in potential negative effects on consumer perception. Other economic research suggests that PMGs can support a mechanism for collusion among the retailers (Corts, 1997). The presence of PMGs by one retailer provides a disincentive to the other retailers to lower their prices because their price will be matched by the PMG-offering retailer. In economics, Gal-Or(1989) argued that warranties are perfect signals only in cases in which the intrinsic attributes of products are neither too clustered nor too widely spaced.

This paper highlights the effects of guarantee commitments, which are different from warranties. Under warranties, consumers can replace or repair goods within the warranty time limit. Under a guarantee, consumers have no other choice but to repair within guarantee time limit. This paper focuses on guarantees because producers can repair goods but not replace them in general in China because the replace requires the enough reasons related to the quality. Actually, in China, rare replaces exist in many industries[1].

Some researchers have studied guarantee commitments in industrial organization theory (Corts, 1997; Nie, 2012). In Nie's recent paper (2012), guarantee commitments under monopoly are analyzed, and some interesting results are obtained. Nie(2012) showed that the guarantee limit time has no effect on demand under complete information and the guarantee limit time is reduced under incomplete information under monopoly. This motivates our further research on guarantee commitments under duopoly in industrial organization research. It is also extremely important to acknowledge the market under the guarantee commitments. All of these topics motivate the theoretical aspect of our work.

In the applied aspect, it is exceedingly important to further consider guarantee commitments in many industries. In a market with guarantee commitments, more information is supplied to the consumers and the market seems more stable. There exist some examples of overdue guarantee yielding bankrupt of the corresponding firms. For example, Shandong Xiaoya Group Co. Ltd, who produced Little Duck

[1] http://money.yzforex.com/a/2012-07-02/13412197382553057.html

Gas Water Heater from 1998 to 2001, quitted the industry of water heater in 2001 because of five years' guarantee commitment (http://www.xiaoyagroup.com.cn/). In general, the guarantee of water heater is 1 year.

This paper is organized as follows: the model is given in Section 2, the model is analyzed in Section 3, and some concluding remarks are presented in the final section.

II. The Model

Assume that there are only two producers in a given industry and the duopoly producers face demands for corresponding goods with high maintenance costs. It is assumed that the guarantee length is exactly T_1 for the first producer and T_2 for the second producer. T_1 and T_2 are both constants in this paper. Namely, the first producer pays the maintenance costs for this product for a length of time T_1 after the consumers buy the product from the first producer. Once the guarantee time limit expires, the consumers have to pay for any maintenance costs. The same situation applies to the second producer, except that the guarantee length is T_2. As an extreme case, $T = 0$ is the extreme case without a commitment to guarantee.

We let p_1 and q_1 denote the price and the quantity sold of goods from the first producer, respectively. Similarly, we define p_2 and q_2 for the second producer.

The cost of each product incurred by production is represented by c_0. Assume that the time until each good needs to be repaired follows the exponential distribution. The density function at time t is then $\varphi(t,\lambda) = \lambda e^{-\lambda t}$ for $t \geq 0$, where λ is a constant dependent on the quality of the goods and λ^{-1} is the average life expectancy of the corresponding goods. The parameter λ depends on the technique of the producers. To simplify the problem, we assume that there are no differences in the technique for two producers. We further assume that $0 \leq T \leq \lambda^{-1}$. The probability to repair a unit of goods between $t = 0$ to $t = t_0$ is $\int_0^{t_0} \varphi(t,\lambda)dt = 1 - e^{-\lambda t_0}$. It is rational that the exponential distribution is employed because the life expectancy of many electronic products satisfy this type of distribution.

We assume that the costs to repair the quantity s of the corresponding goods each time are $c(s) = cs^2$ where $c > 0$ is a constant. We also assume that the utility of the consumers is quasi-linear in the consumption of the goods q and money m. Namely, the quasi-linear utility function is $U(q,m) = u(q) + m$, in which the wealth has the linear effects on the utility. We further assume that u is continuously differentiable. The model for the consumers is given as follows: given any prices $p = (p_1, p_2)$ and guarantee times $T = (T_1, T_2)$, consumers choose $q = (q_1, q_2)$ to maximize their utility. The following utility maximization problem (UMP) is given by

$$\max_{q_1,q_2} \ u(q) = U(q,m) - q_1 p_1 - q_2 p_2 - c[q_1 \int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 - c[q_2 \int_{T_2}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 \ . \tag{1}$$

Let $q(p,T)$ be the static demand functions associated with the utility function U. Denote the revenue and the marginal revenue of the first producer by $R_1(p) = q_1(p_1 - c_0) - c[q_1 \int_0^{T_1} \varphi(t,\lambda)dt]^2$ and $MR_1(p)$, respectively. Given the prices $p = (p_1, p_2)$ and the corresponding guarantee times $T = (T_1, T_2)$, the profits of the first producer are

$$\pi_1(p,T) = q_1(p_1, p_2, T_1, T_2)(p_1 - c_0) - c[q_1(p_1, p_2, T_1, T_2) \int_0^{T_1} \varphi(t,\lambda)dt]^2. \tag{2}$$

Similarly, denote the revenue and the marginal revenue of the second producer by $R_2(p) = q_2(p_2 - c_0) - c[q_2 \int_0^{T_1} \varphi(t,\lambda)dt]^2$ and $MR_2(p)$, respectively. The profits of the second producer are

$$\pi_2(p,T) = q_2(p_1, p_2, T_1, T)(p_2 - c_0) - c[q_2(p_1, p_2, T_1, T_2) \int_0^{T_2} \varphi(t,\lambda)dt]^2. \tag{3}$$

Equations (2) and (3) constitute a Bertrand model. This model is more difficult than the classic Bertrand model because the both producers can simultaneously choose the guarantee time and the prices. To simplify the problem, we also assume that no difference in the quality of the products between two producers exists, which implies that $\frac{\partial^2 u}{\partial q_1^2} = \frac{\partial^2 u}{\partial q_2^2}$ at the equilibrium state. We also note that repair cost function is quadratic in s. In this situation, the competition to repair goods is eliminated to a certain degree, which is different from that in Nie's work (2012). We further point out that we just consider once repair in guarantee time limit. In fact, there are cases with multiple repairs and the probability with twice repairs is very small. We therefore neglect situation with multiple repairs.

In fact, the demand function of an individual consumer may be a discrete function in many industries. The total demand function is also discrete and the total demand is very large in general. When the demand is very large, it is rational to assume that the total demand function (or a utility function) is continuous. It is therefore approximately employed continuous function to model this industry in this work. The following assumptions are given to guarantee that a solution to the above system exists.

Assumptions:

(A) $u(q)$ is concave in q_1 and q_2. Also, $q(p,T) > 0$ for all $p = (p_1, p_2)$ and $T = (T_1, T_2)$.

(B) π_1 is concave in p_1 and T_1. Similarly, π_2 is concave in p_2 and T_2.

Assumptions (A) and (B) guarantee the existence of equilibrium for the system. Furthermore, Assumption (A) ensures the demand is always positive for any prices.

III. Main results

We now consider the equilibrium solution to the above system.

III. I. The solution for the consumers

Under Assumptions (A) and (B), considering the problem of the consumers' point of view with the first order optimal conditions of (1), we have

$$\frac{\partial u}{\partial q_1} = g_1 = \frac{\partial U}{\partial q_1} - p_1 - 2cq_1[\int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 = 0\,, \tag{4}$$

$$\frac{\partial u}{\partial q_2} = g_2 = \frac{\partial U}{\partial q_2} - p_2 - 2cq_2[\int_{T_2}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 = 0\,. \tag{5}$$

Based on the above first-order conditions, we address the relationship between the quantity of the products and the prices.

Proposition 1: Under Assumptions (A)-(B), we have $\frac{\partial q_1}{\partial p_1} < 0, \frac{\partial q_2}{\partial p_2} < 0,$

$$\frac{\partial q_1}{\partial p_2}\begin{cases} > 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} < 0 \\ = 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} = 0 \\ < 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} > 0 \end{cases} \quad \text{and} \quad \frac{\partial q_2}{\partial p_1}\begin{cases} > 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} < 0 \\ = 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} = 0 \\ < 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} > 0 \end{cases} \quad \text{for the equilibrium state.}$$

Proof: See in appendix.

Remarks: The above results illustrate that, if products in both firms have positive externality, higher prices from one producer result in a greater quantity of goods sold by the rival producer. If products of two firms are complementary (under Spence-Mirrlees condition or $\frac{\partial^2 u}{\partial q_1 \partial q_2} \geq 0$), higher prices from one producer yield a lower quantity of goods sold by the rival producer. The classic conclusions, higher prices correspondingly yielding a lower quantity of goods, hold.

We now consider the parameters c and T.

Proposition 2: Under Assumptions (A)-(B), we have $\frac{\partial q_1}{\partial c} < 0$, $\frac{\partial q_2}{\partial c} < 0$,

$$\frac{\partial q_i}{\partial T_i} > 0 \text{ and } \frac{\partial q_i}{\partial T_j}\begin{cases} < 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} < 0 \\ = 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} = 0 \\ > 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} > 0 \end{cases}.$$

Proof: See in appendix.

Note that higher costs cause lower quantity, which is consistent with the classic result in economics. Longer guarantee commitment results in a higher quantity of goods sold. As an extreme case, without guarantee, demand is reduced.

Consider the term $\frac{\partial q_1}{\partial c}$, we have

$$\frac{\partial^2 q_1}{\partial p_1 \partial c} = \frac{\partial(\frac{\partial q_1}{\partial p_1})}{\partial c} = \frac{\partial[-\frac{\frac{\partial g_1}{\partial p_1}}{\frac{\partial g_1}{\partial q_1}}]}{\partial c} = \frac{\partial\{\frac{1}{\frac{\partial^2 U}{\partial q_1{}^2} - 2c[\int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2}\}}{\partial c}$$

$$= \frac{2[\int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2}{\{\frac{\partial^2 U}{\partial q_1{}^2} - 2c[\int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2\}^2}$$

$$= 2(\frac{\partial q_1}{\partial p_1})^2[\int_{T_1}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 > 0. \tag{6}$$

Similarly, we have

$$\frac{\partial^2 q_2}{\partial p_2 \partial c} = 2(\frac{\partial q_2}{\partial p_2})^2[\int_{T_2}^{\lambda^{-1}} \varphi(t,\lambda)dt]^2 . \tag{7}$$

The demand is above discussed. We now consider the producers at the equilibrium state based on the analysis for the consumers.

III. II. The equilibrium for the producers

The optimal solution to (1)—(3) is implicitly given here. The corresponding first-order optimal conditions to (2)—(3) are as follows:

$$\frac{\partial \pi_1}{\partial p_1} = f_1 = q_1 + \frac{\partial q_1}{\partial p_1}\{p_1 - c_0 - 2cq_1[\int_0^{T_1} \varphi(t,\lambda)dt]^2\} = 0 , \tag{8}$$

$$\frac{\partial \pi_2}{\partial p_2} = f_2 = q_2 + \frac{\partial q_2}{\partial p_2}\{p_2 - c_0 - 2cq_2[\int_0^{T_2} \varphi(t,\lambda)dt]^2\} = 0 , \tag{9}$$

$$\frac{\partial \pi_1}{\partial T_1} = f_3 = -2cq_1{}^2 \int_0^{T_1} \varphi(t,\lambda)dt\varphi(T_1,\lambda) + \frac{\partial q_1}{\partial T_1}\{p_1 - c_0 - 2cq_1[\int_0^{T_1} \varphi(t,\lambda)dt]^2\} = 0 , \tag{10}$$

$$\frac{\partial \pi_2}{\partial T_2} = f_4 = -2cq_2{}^2 \int_0^{T_2} \varphi(t,\lambda)dt\varphi(T_2,\lambda) + \frac{\partial q_2}{\partial T_2}\{p_2 - c_0 - 2cq_2[\int_0^{T_2} \varphi(t,\lambda)dt]^2\} = 0 . \tag{11}$$

Under the Assumptions (A)—(B), a unique solution to (8)—(11) exists. Denote the optimal solution to (1)—(3) by $p^* = (p_1^*, p_2^*)$, $T^* = (T_1^*, T_2^*)$ and $\pi^* = (\pi_1^*, \pi_2^*)$ along with $q^* = (q_1^*, q_2^*)$, which is also the solution to the system (8)—(11). Moreover, by symmetry, we have $p_1^* = p_2^*$ $q_1^* = q_2^*$ and $T_1^* = T_2^*$. (2)-(3) and (8)-(11) yield

$$\pi_i^* = -\frac{(q_i^*)^2}{\partial q_i / \partial p_i} + c(q_i^*)^2 [\int_0^{T_i^*} \varphi(t,\lambda)dt]^2 > 0 .$$

(8) and (10) implies

$$\frac{\partial g_1}{\partial T_1}\frac{\partial q_1}{\partial p_1} + \frac{\partial q_1}{\partial T_1}\Big|_{(p^*,T^*)} = 0 . \tag{12}$$

(9) and (11) indicates

$$\frac{\partial g_2}{\partial T_2}\frac{\partial q_2}{\partial p_2} + \frac{\partial q_2}{\partial T_2}\Big|_{(p^*,T^*)} = 0 . \tag{13}$$

In fact, (12) and (13) are also achieved by Proposition 1 and 2. (12) and (13) mean that the effects of guarantee time limit on quantity plus the effects of price on quantity with $\frac{\partial g_1}{\partial T_1}$ equal to zero. This relationship seems interesting.

We now consider the properties of the solutions with the envelope theorem and comparative static analysis.

For the optimal prices to the optimal conditions (8)—(11), we immediately have the following conclusion.

Proposition 3: For the model (1)—(3), $\frac{\partial p_1^*}{\partial c} > 0, \frac{\partial p_2^*}{\partial c} > 0, \frac{\partial T_1^*}{\partial c} < 0$ and $\frac{\partial T_2^*}{\partial c} < 0$.

Proof: See in appendix.

This proposition implies that when technology is improved, meaning that the parameter c decreases, the competition in prices becomes more drastic. The two producers will therefore simultaneously decrease their prices to compete in this duopoly system. Moreover, both producers will extend the guarantee time limit in competition.

Considering the optimal profits, combined with the envelope theorem, the following conclusion holds.

Proposition 4: Under Spence -Mirrlees condition or $\frac{\partial^2 u}{\partial q_1 \partial q_2} \geq 0$ The optimal profits of the producers increase if the parameter c decreases.

Proof: See in appendix.

Remarks: Under Spence-Mirrlees condition or the complementary products of two firms, lower costs incurred by maintenance yields higher profits of firms.

In fact, without Spence-Mirrlees condition , the above conclusions may not hold. Here an example is outlined to illustrate this. Let

$U = A(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - (1-\varepsilon)q_1 q_2$, where $0.5 > \varepsilon > 0$.

In this case, we have $J = \begin{bmatrix} -1 & \varepsilon - 1 \\ \varepsilon - 1 & -1 \end{bmatrix}$, $|J| = 2\varepsilon - \varepsilon^2$ and

$$\frac{\partial q_1}{\partial p_1} = -\frac{\begin{vmatrix} \frac{\partial g_1}{\partial p_1} & \frac{\partial g_1}{\partial q_2} \\ 0 & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|} = -\frac{\begin{vmatrix} -1 & \frac{\partial g_1}{\partial q_2} \\ 0 & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{2\varepsilon - \varepsilon^2} = -\frac{1}{2\varepsilon - \varepsilon^2} < 0 .$$

(8) implies the relationship $p_1^* - c_0 - 2cq_1^*[\int_0^{T_1} \varphi(t,\lambda)dt]^2 \quad = (2\varepsilon - \varepsilon^2)q_1^*$.

In this case, we have $\frac{\partial^2 u}{\partial q_1 \partial q_2} = -1 + \varepsilon$ or $\frac{\partial g_1}{\partial q_2} = -1 + \varepsilon < 0$. Furthermore, from $\pi_i^* = -\frac{(q_i^*)^2}{\partial q_i / \partial p_i} + c(q_i^*)^2[\int_0^{T_i^*} \varphi(t,\lambda)dt]^2 = (2\varepsilon - \varepsilon^2)(q_i^*)^2 + c(q_i^*)^2[\int_0^{T_i^*} \varphi(t,\lambda)dt]^2$, we have

$$\frac{\partial \pi_1^*}{\partial c} = (\frac{\frac{\partial g_2}{\partial T_2}\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial T_2^*}{\partial c} - \frac{\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial p_2^*}{\partial c} - \frac{\frac{\partial g_1}{\partial c}\frac{\partial g_2}{\partial q_2} - \frac{\partial g_2}{\partial c}\frac{\partial g_1}{\partial q_2}}{|J|})\{p_1^* - c_0 - 2cq_1^*[\int_0^{T_1} \varphi(t,\lambda)dt]^2\}$$
$$+(q_1^*)^2[\int_0^{T_1} \varphi(t,\lambda)dt]^2$$
$$= q_1^*[(1-\varepsilon)\frac{\partial p_2^*}{\partial c} + (1-\varepsilon)\frac{\partial g_2}{\partial T_2}\frac{\partial T_2^*}{\partial c} - \frac{\partial g_1}{\partial c} + (1-\varepsilon)\frac{\partial g_2}{\partial c}] + (q_1^*)^2[\int_0^{T_1} \varphi(t,\lambda)dt]^2.$$

In the above formulation, under $\varepsilon \to 0$, if $\frac{\partial p_2^*}{\partial c}$ and $\frac{\partial g_2}{\partial T_2}\frac{\partial T_2^*}{\partial c}$ are all large enough,

$$\frac{\partial \pi_1^*}{\partial c} = q_1^*[(1-\varepsilon)\frac{\partial p_2^*}{\partial c} + (1-\varepsilon)\frac{\partial g_2}{\partial T_2}\frac{\partial T_2^*}{\partial c} - \frac{\partial g_1}{\partial c} + (1-\varepsilon)\frac{\partial g_2}{\partial c}] + (q_1^*)^2[\int_0^{T_1} \varphi(t,\lambda)dt]^2 > 0.$$

The parameter c decreasing means the improvement of the maintenance technique or the management, which yields the reduction of the profits of the producers. This is in reality a surprising conclusion. However, this is rational if an overly long guarantee commitment is launched, which happens in many examples in household appliance industries in China. For example, Shandong Xiaoya Group Co. Ltd with Little Duck Gas Water Heater from 1998 to 2001, quitted the industry of water heater in 2001 because of five years' guarantee commitment. These companies give a guarantee commitment that is much longer than optimal, resulting in decreased profits when the technology in the corresponding industry is improved. This results in the bankruptcy of these companies. Furthermore, this conclusion explains the lower profits of many household appliance industries because of excessive competition.

Moreover, Zwetsloot, Aaltonen, Wybo et al. (2013) concluded that the commitment strategies improve the industry safety, while this paper argued that the guarantee commitment reduces the industrial safety. The contrary conclusions are achieved because of the different commitments.

IV. Concluding remarks

In this paper, we analyzed guarantee commitments under a duopoly market structure. We analyzed the properties of a market defined by the Bertrand game model, which is more complex than the classic Bertrand model. Surprisingly, we found that technological innovation may reduce the profits and the level of competition. This result rationally explains many examples, such as the household appliance industry in China, where improved technology has led to lower profits.

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Appendix

Proof of Proposition 1

Proof: Considering (4), we have $\frac{\partial g_1}{\partial q_1} < 0$ according to Assumption (A), or the concavity of the function $u(q)$. The Jacobian matrix of (4) and (5) is $J = \begin{bmatrix} \frac{\partial g_1}{\partial q_1} & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial q_1} & \frac{\partial g_2}{\partial q_2} \end{bmatrix}$ and $|J| > 0$ by the hypothesis of concavity. We also get $\frac{\partial g_1}{\partial p_1} = -1$ from (4). We therefore have $\frac{\partial q_1}{\partial p_1} = -\frac{\begin{vmatrix} \frac{\partial g_1}{\partial p_1} & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial p_1} & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|} = -\frac{\begin{vmatrix} \frac{\partial g_1}{\partial p_1} & \frac{\partial g_1}{\partial q_2} \\ 0 & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|} < 0$.

Similarly, by (5), we have $\frac{\partial g_2}{\partial q_2} < 0$. We further obtain $\frac{\partial q_2}{\partial p_2} < 0$.

Here we consider $\frac{\partial q_1}{\partial p_2}$. According to the implicit function theorem, there exists the unique solution that is differentiable. Furthermore, by virtue of $\frac{\partial g_1}{\partial q_2} = \frac{\partial g_2}{\partial q_1} = \frac{\partial^2 U}{\partial q_1 \partial q_2}$,we have

$$\frac{\partial q_1}{\partial p_2} = -\frac{\begin{vmatrix} \frac{\partial g_1}{\partial p_2} & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial p_2} & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|} = -\frac{\begin{vmatrix} 0 & \frac{\partial g_1}{\partial q_2} \\ -1 & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|} \begin{cases} > 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} < 0 \\ = 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} = 0 \\ < 0 & \frac{\partial^2 u}{\partial q_1 \partial q_2} > 0 \end{cases}$$

We therefore have $\frac{\partial q_1}{\partial p_2}\begin{cases} >0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}<0 \\ =0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}=0 \\ <0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}>0 \end{cases}$. Similarly, we have $\frac{\partial q_2}{\partial p_1}\begin{cases} >0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}<0 \\ =0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}=0 \\ <0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}>0 \end{cases}$.

This completes the proof.

Proof of Proposition 2

Proof: We first show that $\frac{\partial q_1}{\partial c}<0$. We have $\frac{\partial g_1}{\partial q_1}<0$ from Assumption (A), because $\frac{\partial g_1}{\partial c}=-2q_1[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2<0$. By (4), we have $\frac{\partial q_1}{\partial c}<0$. Similarly, we see that (5) implies $\frac{\partial q_2}{\partial c}<0$.

Next, $\frac{\partial g_1}{\partial T_1}=-4cq_1[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt][-\varphi(T_1,\lambda)]>0$ from (4). We therefore have

$$\frac{\partial q_1}{\partial T_1}=-\frac{\partial q_1}{\partial T_2}-\frac{\begin{vmatrix} \frac{\partial g_1}{\partial T_1} & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial T_1} & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|}=-\frac{\begin{vmatrix} \frac{\partial g_1}{\partial T_1} & \frac{\partial g_1}{\partial q_2} \\ 0 & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|}>0$$

. Similarly, (5) implies $\frac{\partial q_2}{\partial T_2}>0$.

$$\frac{\partial q_1}{\partial T_2}-\frac{\begin{vmatrix} \frac{\partial g_1}{\partial T_2} & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial T_2} & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|}=-\frac{\begin{vmatrix} 0 & \frac{\partial g_1}{\partial q_2} \\ \frac{\partial g_2}{\partial T_2} & \frac{\partial g_2}{\partial q_2} \end{vmatrix}}{|J|}\begin{cases} <0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}<0 \\ =0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}=0 \\ >0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}>0 \end{cases}$$

. Similarly, we have

$$\frac{\partial q_2}{\partial T_1}-\frac{\begin{vmatrix} \frac{\partial g_1}{\partial q_1} & \frac{\partial g_1}{\partial T_1} \\ \frac{\partial g_2}{\partial q_1} & \frac{\partial g_2}{\partial T_1} \end{vmatrix}}{|J|}=-\frac{\begin{vmatrix} \frac{\partial g_1}{\partial q_1} & \frac{\partial g_1}{\partial T_1} \\ \frac{\partial g_2}{\partial q_1} & 0 \end{vmatrix}}{|J|}\begin{cases} <0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}<0 \\ =0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}=0 \\ >0 & \frac{\partial^2 u}{\partial q_1 \partial q_2}>0 \end{cases}$$

. This completes the proof.

Proof of Proposition 3

First, we show $q_1^* + c\dfrac{\partial q_1^*}{\partial c}$ >0.

$$q_1^* + c\frac{\partial q_1^*}{\partial c} = q_1^* - c\frac{\frac{\partial g_1}{\partial c}}{\frac{\partial g_1}{\partial q_1^*}} = q_1^* - \frac{-2cq_1^*[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2}{\frac{\partial^2 U}{\partial q_1^2} - 2c[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2} > q_1^* - \frac{-2cq_1^*[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2}{-2c[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2} = 0\,.$$

The first inequality comes from the fact that $\dfrac{\partial^2 U}{\partial q_1^2}$ <0. Considering (8) and Assumptions (A)—(B), we get $\dfrac{\partial f_1}{\partial p_1^*} < 0$ and

$$\begin{aligned}
\frac{\partial f_1}{\partial c} &= \frac{\partial q_1^*}{\partial c} + \frac{\partial^2 q_1^*}{\partial p_1^* \partial c}\{p_1^* - c_0 - 2cq_1[\int_0^{T_1}\varphi(t,\lambda)dt]^2\} \\
&\quad - 2\frac{\partial q_1^*}{\partial p_1^*}q_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2 - 2\frac{\partial q_1^*}{\partial p_1^*}c[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}\frac{\partial q_1^*}{\partial c} \\
&= \frac{\partial q_1^*}{\partial c} + (\frac{\partial q_1^*}{\partial p_1^*})^2 2[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2\{-q_1^* / \frac{\partial q_1^*}{\partial p_1^*}\} - 2\frac{\partial q_1^*}{\partial p_1^*}q_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2 - 2\frac{\partial q_1^*}{\partial p_1^*}c[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}\frac{\partial q_1^*}{\partial c} \\
&= \frac{\partial q_1^*}{\partial c} - 2\frac{\partial q_1^*}{\partial p_1^*}[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2 q_1^* - 2\frac{\partial q_1^*}{\partial p_1^*}q_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2 - 2\frac{\partial q_1^*}{\partial p_1^*}c[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}\frac{\partial q_1^*}{\partial c} \\
&= -2\frac{\partial q_1^*}{\partial p_1^*}q_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2 - 2\frac{\partial q_1^*}{\partial p_1^*}c[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}\frac{\partial q_1^*}{\partial c} \\
&= -2\frac{\partial q_1^*}{\partial p_1^*}[\int_0^{T_1}\varphi(t,\lambda)dt]^2(q_1^* + c\frac{\partial q_1^*}{\partial c}) > 0.
\end{aligned}$$

From the proof of Propositions 1 and 2, we have $\dfrac{\partial q_1^*}{\partial c} = 2\dfrac{\partial q_1^*}{\partial p_1^*}[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2 p_1^*$.

Therefore, $\dfrac{\partial q_1^*}{\partial c} + \dfrac{\partial^2 q_1^*}{\partial p_1^* \partial c}\{p_1^* - c_0 - 2cq_1[\int_0^{T_1}\varphi(t,\lambda)dt]^2\} = 0$. The last inequality is obtained from $\dfrac{\partial q_1^*}{\partial p_1^*}$ <0 and $q_1^* + c\dfrac{\partial q_1^*}{\partial c}$ >0. According to the implicit function theorem, we get $\dfrac{\partial p_1^*}{\partial c} > 0$. Similarly, considering (9), we have $\dfrac{\partial p_2^*}{\partial c} > 0$.

Similarly to the above analysis, we also have

$\frac{\partial q_1^*}{\partial T_1^*}+\frac{\partial^2 q_1^*}{\partial p_1^* \partial T_1^*}\{p_1^*-c_0-2cq_1[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}$ =0. We also show that

$[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\partial q_1^*}{\partial T_1^*}+2q_1^*\varphi(T_1,\lambda)$ >0.

$$[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\partial q_1^*}{\partial T_1^*}+2q_1^*\varphi(T_1,\lambda)=-[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\frac{\partial g_1}{\partial T_1}}{\frac{\partial g_1}{\partial q_1}}+2q_1^*\varphi(T_1,\lambda)$$

$$=-[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{4cq_1^*\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt\varphi(T_1,\lambda)}{\frac{\partial^2 U}{\partial q_1^2}-2c[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2}+2q_1^*\varphi(T_1,\lambda)$$

$$>2q_1^*\varphi(T_1,\lambda)+[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{4cq_1^*\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt\varphi(T_1,\lambda)}{2c[\int_{T_1}^{\lambda^{-1}}\varphi(t,\lambda)dt]^2}>0$$

The first inequality comes from $\frac{\partial^2 U}{\partial q_1^2}$ <0. The last inequality is obvious. Furthermore, the formulation $\frac{\partial f_1}{\partial c}$ >0 holds.

$$\begin{aligned}\frac{\partial f_1}{\partial T_1}&=\frac{\partial q_1^*}{\partial T_1^*}+\frac{\partial^2 q_1^*}{\partial p_1^* \partial T_1^*}\{p_1^*-c_0-2cq_1[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}\\&\quad-2c[\int_0^{T_1}\varphi(t,\lambda)dt]^2\frac{\partial q_1^*}{\partial p_1^*}\frac{\partial q_1^*}{\partial T_1^*}-4cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\partial q_1^*}{\partial p_1^*}\varphi(T_1,\lambda)\\&=-2c[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\partial q_1^*}{\partial p_1^*}\{[\int_0^{T_1}\varphi(t,\lambda)dt]\frac{\partial q_1^*}{\partial T_1^*}+2q_1^*\varphi(T_1,\lambda)\}>0.\end{aligned}$$

We therefore have $\frac{\partial T_1^*}{\partial c}<0$. Similarly, considering (9), we have $\frac{\partial T_2^*}{\partial c}<0$. This completes the proof.

Proof of Proposition 4

Proof. The result is shown by envelope theorem. We first consider π_1 and the following conclusion holds.

$$\frac{\partial \pi_1^*}{\partial c}=\frac{\partial \pi_1^*}{\partial p_2^*}\frac{\partial p_1^*}{\partial c}+\frac{\partial \pi_1^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial c}+\frac{\partial \pi_1^*}{\partial T_1^*}\frac{\partial T_1^*}{\partial c}+\frac{\partial \pi_1^*}{\partial T_2^*}\frac{\partial T_2^*}{\partial c}+\frac{\partial \pi_1^*}{\partial c}$$

$$=\frac{\partial \pi_1^*}{\partial p_2^*}\frac{\partial p_1^*}{\partial c}+\frac{\partial \pi_1^*}{\partial T_2^*}\frac{\partial T_2^*}{\partial c}+\frac{\partial \pi_1^*}{\partial c}$$

$$=(\frac{\partial q_1^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial c}+\frac{\partial q_1^*}{\partial T_2^*}\frac{\partial T_2^*}{\partial c}+\frac{\partial q_1^*}{\partial c})\{p_1^*-c_0-2cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}-(q_1^*)^2[\int_0^{T_1}\varphi(t,\lambda)dt]^2.$$

If $\frac{\partial^2 u}{\partial q_1 \partial q_2}=0$, we have

$$\frac{\partial \pi_1^*}{\partial c}=\frac{\partial q_1^*}{\partial c}\{p_1^*-c_0-2cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}-(q_1^*)^2[\int_0^{T_1}\varphi(t,\lambda)dt]^2<0.$$

If $\frac{\partial^2 u}{\partial q_1 \partial q_2}\neq 0$, we have

$$\frac{\partial \pi_1^*}{\partial c}=(\frac{\partial q_1^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial c}+\frac{\partial q_1^*}{\partial T_2^*}\frac{\partial T_2^*}{\partial c}+\frac{\partial q_1^*}{\partial c})\{p_1^*-c_0-2cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}-(q_1^*)^2[\int_0^{T_1}\varphi(t,\lambda)dt]^2$$

$$=(-\frac{\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial p_2^*}{\partial c}+\frac{\frac{\partial g_2}{\partial T_2}\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial T_2^*}{\partial c}-\frac{\frac{\partial g_1}{\partial c}\frac{\partial g_2}{\partial q_2}-\frac{\partial g_2}{\partial c}\frac{\partial g_1}{\partial q_2}}{|J|})\{p_1^*-c_0-2cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}-(q_1^*)^2[\int_0^{T_1}\varphi(t,\lambda)dt]^2.$$

If $\frac{\partial^2 u}{\partial q_1 \partial q_2}>0$, we have $\frac{\partial g_1}{\partial q_2}>0$. Therefore, by virtue of Proposition 3, we achieve the following conclusion.

$$\frac{\partial \pi_1^*}{\partial c}=(-\frac{\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial p_2^*}{\partial c}+\frac{\frac{\partial g_2}{\partial T_2}\frac{\partial g_1}{\partial q_2}}{|J|}\frac{\partial T_2^*}{\partial c}+\frac{\partial q_1^*}{\partial c})\{p_1^*-c_0-2cq_1^*[\int_0^{T_1}\varphi(t,\lambda)dt]^2\}-(q_1^*)^2[\int_0^{T_1}\varphi(t,\lambda)dt]^2<0.$$

Similarly, we have $\frac{\partial \pi_2^*}{\partial c}<0$. Therefore, under Spence-Mirrlees condition, we have $\frac{\partial \pi_i^*}{\partial c}<0$. This completes the proof.